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PROBLEM MODELING METHODS IN MATHEMATICS EDUCATION

Abstract

Solving problems using different methods has always been a topical issue in the teaching of mathematics. Depending on the structure of the problem, mathematical models are widely used for students to visually master the relationships between given and sought quantities. The concept of the model in solving mathematical problems, its applications and forms have always been of interest to researchers in this field. This article considers the theoretical and practical bases of problem modeling, the requirements for them, the conditions under which the model is given, the possibility of using several models in solving a problem. Here, examples of solving specific problems related to content lines are considered, similarities and differences of these content lines in the modeling are analyzed.

Keywords: *problem, quantity, modeling, table, methods of solving, algorithm*

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Riyaziyyat təlimində məsələnin modelləşdirilməsi üsulları

Xülasə

Riyaziyyatın tədrisində məsələnin müxtəlif üsulların tətbiqi ilə həll edilməsi həmişə aktual mövzulardan biri olub. Məsələnin strukturundan asılı olaraq verilən və axtarılan kəmiyyətlər arasındakı asılılıqların şagirdlər tərəfindən əyani mənimsənilməsi üçün riyazi modellərdən geniş istifadə olunur. Riyazi məsələ həllində model anlayışı, onun tətbiqləri və formaları bu sahə ilə məşğul olan tədqiqatçıları həmişə maraqlandırmışdır. Bu məqalədə məsələnin modelləşdirilməsinin nəzəri və praktik əsasları, onlara verilən tələblər, modelin hansı şərt daxilində verilməsi, bir məsələnin həllində bir neçə modeldən istifadənin imkanları nəzərdən keçirilmişdir. Burada məzmun xətlərinə aid konkret məsələlərin həlli nümunələrinə baxılmış, modelləşdirmənin aparılmasında bu məzmun xətlərinin oxşar və fərqli əlamətləri təhlil olunmuşdur.

Açar sözlər: *məsələ, kəmiyyət, modelləşdirmə, cədvəl, həll üsulu, alqoritm*

Introduction

Problem solving is considered an indispensable tool in mathematics education. Mastering the content of the case by the students, analyzing the data and the dependencies between them can be interpreted in different ways. When solving a mathematical problem, it is widely used in building models of the objects given in the problem to visualize the dependencies between the quantities given in the problem and to determine the quantity sought.

Thus, when analyzing the numerical values of the quantities given in the content of the problem and the functional dependencies between them, sometimes there is a need to look at the general combination of certain images for their understanding. It is known that psychological, general pedagogical and specific theories defining the learning process constantly complement each other. Different training theories can be created based on different psychological aspects of training. The possibility of having different theories in training creates conditions for the emergence of new theories.

The organization of new theories in mathematics education may be related to aspects such as application of mathematics, logic in mathematics, etc. Regardless of the form of the theory, the innovation applied in training stimulates the development and direction of training. Due to the development of science and technology, a wide range of mathematical methods and their applications have been used in recent years related to people's life activities. The main goal of the proposed new methods and approaches is to improve the theoretical, practical and methodical issues of problem solving training.

Research and results. The main content of mathematical problems is related to the practical activities of people. This includes sections such as mathematical economics, mathematical chemistry, mathematical physics, mathematical linguistics, etc. Each of them is a certain mathematical model and is used to determine the relations of relevant objects and events. Creating and describing each object, event, quantity or relationships between them in sequence is called modeling. Model is derived from the Latin word «modulus», which means measure, pattern, defined rule. The concept of model has a special place in the study of every science. Ensuring the independent activity of students in the training process depends on many conditions. In order for the mathematical knowledge acquired by students in the secondary school course to be stable and long-term, it is important to carry out the descriptions and modeling based on logic. A brief description of the content of the case and the correct construction of models such as pictures, schemes, drawings, etc. to illustrate the situations described in it provide an opportunity for the development of the students' future activities. Special importance was given to the models proposed during problem solving in the mathematics subject curriculum and different examples of the models applied in problem solving were offered in the textbooks and methodical materials intended for teachers (Gahramanova, 2022; Gahramanova, 2023; Mammadov, 2018).

The modeling method is widely used in solving mathematical problems. Animating and creating the real state of any problem or situation on the basis of a special logical description can be considered as mathematical modeling. A mathematical model is formed as a result of observation in the detection of object and event, relations (dependencies) between them. The model may consist of several stages:

– transforming the model of the proposed problem (situation) into a mathematical theory. It is also called the formation and construction of a mathematical model;

– solution of the problem within the framework of mathematical theory (internal solution model);

– conversion of the result obtained from the solution of the mathematical problem into the language in which the problem was given before (interpretation of the solution).

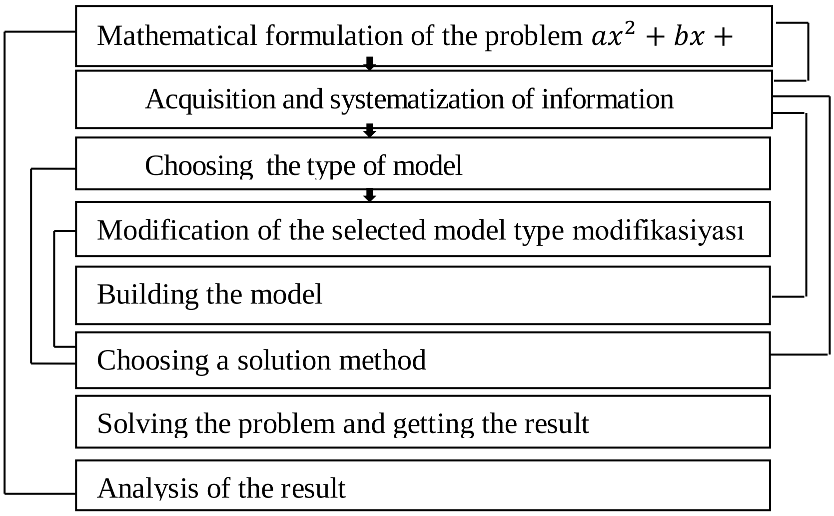
The object and event model can be described in several ways. The dependencies between the components of the model depend on the realization of the state of the object and the event. When the functional dependencies between the quantities are applied to the elements of the "Geometry" content line, the transformation (change) of the figures within the condition should be taken into account in the mathematical expression of the dependencies. These cases are theoretically and practically examined in detail (Asadov, 2020: 254-263).

Modeling should be done according to the structure of content lines. The processes in the modeling of the "Geometry", "Algebra and functions", "Measurements", etc. content lines are also applied when solving problems of mechanics, physics, chemistry, geography, etc. with practical content.

Whole-part model, picture, drawing, graph, diagram are widely used in mastering the content of textual issues. Mathematical modeling plays a key role in understanding complex dynamic systems and summarizing functional dependencies between its elements. In modeling the geosystem, the natural object is artificial.

It is practically impossible to model all the processes of a complex system. However, in modeling, it is important to take into account the main properties of the object and take into account the internal and external relations between its elements. Quantitative indicators have a special place in the internal and external relations of the system. In the analysis of the studied object, the modeling method is evaluated not only as a quantitative indicator, but also as a qualitative criterion. The direction of learning and studying complex geosystems during the improvement of the model of the object is mainly based on the mathematical model. Blocks and elements of any wide system can be involved in geosystem modeling. The construction of models in solving mathematical problems and complex problems related to other disciplines is governed by theoretical and practical principles. Using a mathematical model to solve problems related to practical life makes it possible to reveal interdisciplinary relationships. For example, the use of mathematical modeling in solving complex problems related to geography simplifies the solution of the problem. Mathematical-geographical modeling is a complex process and consists of several successive stages. Here, the problems such as the structure and relief of the Earth, industry, agriculture, population growth, etc. are considered to be studied.

The scheme of mathematical modeling of the geographical object was given by academician Y.P.Arkhipov as follows (Arkhipov, Blazhko, 1976: 39).



Scheme 1.

Here, each of the stages differs partially depending on the content of the problem. So, these stages are performed in the analysis of any problem. The stages of modeling will be changed from the analysis of the result of the problem to the problem statement. In this case, not one, but several models can be used to determine the unknown quantity. The result of the first model can be considered as the first stage for the next model.

Mathematical problems solved in the secondary school course are modeled in one way or another. Modeling in biology, medicine, and economics is based on mathematical modeling even though it is intended for a real object. In the construction of all models, it can be divided into two parts, depending on the scheme and the signs (symbols) in the field to which it belongs.

Modeling according to the scheme can be provided by the object itself or by its description defining its graphical dependence. For example, when modeling the situation in textual issues related to action, the situation corresponding to the question of the issue should be clearly described in the model.

Modeling of other non-essential cases can be done verbally according to the content of the problem. Sometimes, all special cases of complex text problems, including plane and space problems related to the "Geometry" content line, are taken into account in the modeling. The simple and understandable state of the model has a positive effect on the interest and mastery of students.

Some researchers estimate that choosing the right model in solving a mathematical problem is about half of solving the problem (Kolyagin, 1977).

The graphic model is widely used in the schematic generalization of the situation. Schematic modeling refers to concepts such as a picture, a conventional picture, a drawing, a simple scheme. In these processes, except for some cases, the dependencies between the quantities in the modeling and their numerical values cannot be intuitively accepted as exact solutions based on the description. Every calculation on the basis of modeling should be based on purely mathematical laws. For example, marking the properties of a quadratic function based on its graph can be considered a correct judgment. Based on the description, the analytical expression of the function can be written. Therefore, the quantities given in the problem and the dependencies between them can be converted into ordinary language and mathematical language with the help of certain symbols. A short summary of the content of the text problem can also be taken as a model. If the number and dependence of the quantities involved in the problem is big, it is also possible to express it in the form of a table. The transformation of the content of the problem into mathematical language and its modeling can be considered as the initial stage. If this stage is properly organized from a pedagogical and psychological point of view, there will be no difficulty in opening the content based on the description considered in the second stage. Marking the properties of the function based on the graph, determining the value of the quantities based on the given diagram, analyzing and discussing the data based on the statistical results can be considered as the second stage of modeling. Modeling of textual problems with symbols is understood as expression, equation, system of equations, inequality, writing of the problem based on operations, etc. In case modeling, schematization, i.e. certain representations (graphs, tables, diagrams, etc.) are usually called auxiliary elements of modeling. And the modeling process that translates into mathematical language is considered as solution modeling. In modeling some issues, it is more efficient to use a segment or a table. Let’s consider the following problem.

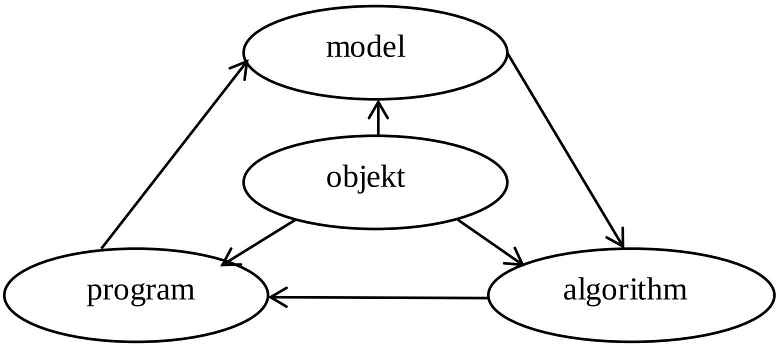
The problem. The three rolls have a total of 168 meters of fabric. After selling 12 meters of fabric from the first roll, 24 meters from the second roll, and 18 meters from the third roll, the same amount of fabric remained in all three rolls. How long fabrics were in each roll initially?

Using "segment" in modeling this problem is closer to visualization. The theoretical materials of modeling in the secondary school mathematics course are mainly formed in the learning process. Implementing modeling based on purely theoretical material can be dangerous in some cases. For example, technical, environmental and economic, etc. theoretical modeling in the complete and accurate study of systems can lead to wrong results. In mathematical modeling, the exact description of the object and the plan of action are given in a certain order. It can be conventionally divided into three stages:

1. Model
2. Algorithm
3. Program

These stages can be described as in scheme 2.

At the first stage, an object "equivalent" to the object is created. The mathematical form of the object, the properties and regularities of the object, and the relationship between its elements are taken into account. The mathematical model is studied by the theoretical method and at this stage additional knowledge about the object is obtained. In the second stage, the selection of the algorithm for the realization of the model on the computer is envisaged.



Scheme 2.

The transformation of the model and algorithm into a computer language, i.e. the creation of a program, constitutes the third stage (Mammadov, 2018). Modeling mathematical problems in the secondary school course is a certain part of the abovementioned stages. Depending on the content of the case, some of these stages are not fully implemented. Thus, understanding the content of the problem, its structure, dependencies between quantities, properties of the object, etc. it is important to build a model so that concepts are mastered by students.

Some problems can be solved with a two-stage whole-part model. The whole-part model can be implemented in various forms. For example, pie chart, adopted uniform segment, rectangular shape, etc. The model can be built in any way that is relevant to the content of the case and effective for the students. The main goal is the student's understanding of the model. The model is built according to the initial data based on the content of the problem. Mathematical operations are performed starting from the last stage of the model. Since the whole-part model is fully described, the main goal here is to find the quantities sought in the problem.

It is not always correct to use the whole-part model when solving a problem. Thus, the method of using a model or solving without a model during the solution of the same problem depends on the stage of training and the age and knowledge levels of the students.

Some time is spent on building a model of the situation described in the problem. Experience shows that building a model corresponding to the content of the problem and designing the problem based on the given model are inseparable processes. However, while considering them separately, some points should not be forgotten. Thus, since one of the numerical values of the quantity given during modeling in two stages is expressed as a fractional number, after accepting it as a whole and building a new model, a problem arises in connection with the previous model. Therefore, theoretical materials should be taken into account when giving these types of questions. After the topics of decimal fractions, comparison of decimal fractions, rounding of decimal fractions and arithmetic operations on decimal fractions are given in the textbook, the elements of the geometry content line are included. The following information about "congruent figures" was given in the textbook.

The concept of "congruent figures" is included here. Figures having the same shape and the same size are called congruent figures. Corresponding elements of quantities can be represented by numbers with different names. When congruent figures are superimposed, all their points coincide. These figures are also called equal figures. To show the congruent sides and angles of figures, small lines are placed on them (Kolmogorov, 1982: 106). The expressions "figures are equal" and "figures are congruent" are different concepts. For example, it can be said that a segment AB is equal to a segment CD (congruent). Because the equality of length is considered here. The final result depends on one (single) measure. However, the statement "triangle ABC is congruent to triangle $A_1B_1C_1$" and "triangle ABC is congruent to triangle A1B1C1" are different in content and may not be true in general. Any figure consists of a certain set of points.

First, we note that the figures A, B, C, D, and E are congruent and when the figures are "superimposed" they overlap. As we know, it is not always possible to "superimpose" any figure. Academician A.N.Kolmogorov explained what it means to "superimpose the figures" in the following way: In geometry, this expression cannot be taken literally. After all, for us, figures are a set of points, and they cannot be "moved" from their place on the plane. Instead of "imposing the figures", their reflection with each other should be considered (Kolmogorov, Cherkasov, 1979).

Problem 3: Prove that the sum of the distances from an arbitrary point located in the interior of an equilateral triangle to its sides is equal to the height of this triangle (Mardanov, Mirzayev, 2004: 60).

In order to prove the given proposal, conditions should be created for students to repeat the theoretical materials related to various topics related to the content line. What are the circumstances mentioned for this problem?

- triangle and its types;
- the main elements of the triangle and their properties;
- internal and external areas, the conditions defining them;
- the area of the triangle.

The situation described in the problem can be visualized geometrically as follows.

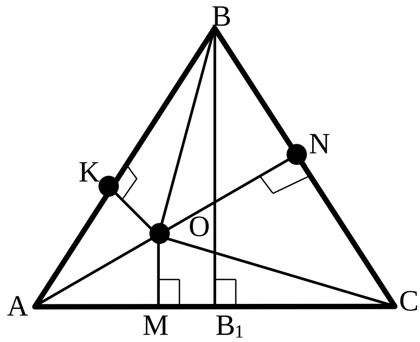


Figure 1.

Let us assume the following notations according to Figure 1 and the condition of the problem.

- In ΔABC, $AB = BC = AC$;
- $OK \perp AB$; $ON \perp BC$, $OM \perp AC$ və $OK = h_1$, $ON = h_2$, $OM = h_3$; $BB_1 \perp AC$, və $BB_1 = h$ olsun.

The area of triangle ABC according to the condition will be $S = \frac{1}{2} \cdot AC \cdot h$.

$$S_{\Delta ABC} = S_{\Delta ABO} + S_{\Delta BOC} + S_{\Delta AOC} \quad (1)$$

If we substitute the notations mentioned in equation (1), we get the following;

$$\frac{1}{2} \cdot AC \cdot h = \frac{1}{2} AB \cdot h_1 + \frac{1}{2} \cdot BC \cdot h_2 + \frac{1}{2} \cdot AC \cdot h_3 \quad (2)$$

If in equation (2), $AB = BC = AC$, then $h = h_1 + h_2 + h_3$. This means that the requested proposal is correct.

Conclusion

Thus, the proposed methods for the application of models in problem solving, the introduction of models in solving complex problems, the realization of intradisciplinary and interdisciplinary models and the construction of relevant models related to different content lines were reviewed from a theoretical and practical point of view.

Scientific novelty of the article. The theoretical, practical and methodical issues of problem modeling have been examined in the article. Here, the construction of the intradisciplinary and interdisciplinary model, the role of training methods in the construction of the model in stages and the classification of models corresponding to the content lines have been studies.

Practical significance and application of the article. The construction of different models of dependences given in the problem enables the development of students' mathematical and logical thinking. Application of acquired knowledge and skills in practice, proposed methods and approaches in studying theoretical and practical materials are appropriate tu use in secondary school mathematics course.

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